Therefore, the values found for the electrodynamic and thermal characteristics of a biconical cavity permit the determination of the greatest achievable level of microwave working power, the thermal rupture modes of the system, and also the correction to the magnitude of the cavity field because of the intrinsic fluctuating thermal radiation of the heated walls. The method proposed for the computation of the electromagnetic and thermal fields of a biconical cavity by using its partition into an approximate profile of elementary inhomogeneities affords the possibility of finding the designated characteristics of a number of microwave units with arbitrary shape of the functional elements.

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UNSTEADY HEAT LOSSES OF UNDERGROUND PIPELINES

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Analytic expressions are presented for the unsteady temperature distribution of the ground and heat losses of an underground pipeline for an arbitrary variation of the temperature of the medium being transferred and boundary conditions of the third kind at the pipe wall and the surface of the ground.

1. The design and operation of oil and gas pipelines require calculating the heat losses of a pipeline under unsteady heat-transfer conditions. Transient thermal processes arise in oil and gas pipelines in turning off oil heating stations and devices for air cooling of gas, stopping the transfer, starting up the pipeline, etc. These processes lower the performance of the system, increase the power expended, and may lead to fusion of the rust-inhibiting insulation, a loss of longitudinal stability, and emergency stopping of transfer. To develop recommendations for ensuring reliable operation of gas and oil pipelines it is necessary to have available relations for calculating unsteady heat losses of pipelines.

Solutions of the problem of unsteady heat transfer between an underground pipeline and the surrounding medium have been obtained under a number of simplifying assumptions. A correlation of the papers on this problem is given in [1]. The most general result for large-diameter pipelines not far below the surface of the ground was obtained in [2]. However, the solution is given in the form of a double sum over eigenfunctions, which complicates its

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practical employment. In addition, this solution converges slowly at the boundaries of the ground and does not take account of its natural temperature distribution. The present paper extends the work in [2] by performing additional investigations of the problem.

2. To find unsteady heat losses of underground pipelines we consider the solution of the heat-conduction equation for the ground

$$\frac{\partial T}{\partial t} = a_{gr}\left(\frac{\partial^2 T_{gr}}{\partial x^2} + \frac{\partial^2 T_{gr}}{\partial y^2}\right) \tag{1}$$

for the following boundary and initial conditions:

$$\partial T_{gr}/\partial n = -\alpha_1 (T - T_{gr})/\lambda_{gr} \quad \text{at} \quad r = R_0; \tag{2}$$

$$\partial T_{gr}/\partial n = \alpha_{so}(T_{gr} - T_{ae})/\lambda_{gr} \quad \text{at} \quad y = 0; \tag{3}$$

$$\lambda_{gr}\frac{\partial T_{gr}}{\partial x} = 0 \quad \text{at} \quad x = 0 \begin{cases} 0 \leqslant y \leqslant h_0 - R_0 \\ h_0 - R_0 \leqslant y < \infty \end{cases}; \tag{4}$$

$$T_{gr} = T_{nat} \quad \text{as} \quad x \to \infty,\ y \to \infty; \tag{5}$$

$$T_{gr} = T_{gro} \quad \text{for} \quad t \leqslant 0. \tag{6}$$

The validity of the assumptions used in writing Eqs. (1)-(6) is confirmed by experimental studies on trunk pipelines and tests on models [3]:

a) The ground is considered as a quasiuniform solid to which the equivalent thermal conductivity model [4] is applicable;

b) the thermophysical properties of the ground are practically temperature independent in the range of transfer parameters considered;

c) the heat flux transmitted by the medium along the pipeline is negligible in comparison with the heat flux in the transverse direction;

d) the pipeline is at a constant depth below the surface of the ground.

In writing the boundary condition at the surface of the ground (3) the effect of solar radiation and reflected radiant flux is taken into account as in [4] by introducing a generalized coefficient of heat transfer α_{so} from the surface of the ground to the atmosphere, and an equivalent air temperature T_{ae}.

3. Since Eq. (1) is linear we seek its solution as a sum of the natural temperature distribution of the ground T_{nat} and the thermal perturbation T_{pi} caused by the pipeline:

$$T_{gr} = T_{nat} + T_{pi}. \tag{7}$$

The function $T_{pi} = T_{pi}(x, y, t)$ is the solution of the equation

$$\frac{\partial T_{pi}}{\partial t} = a_{gr}\left(\frac{\partial^2 T_{pi}}{\partial x^2} + \frac{\partial^2 T_{pi}}{\partial y^2}\right) \tag{1a}$$

with the boundary conditions

$$\partial T_{pi}/\partial n = -\alpha_1 (T - T_{pi} - T_{nat})/\lambda_{gr} - \partial T_{nat}/\partial n \quad \text{at} \quad r = R_0; \tag{2a}$$

$$\partial T_{pi}/\partial y = \alpha_{so} T_{pi}/\lambda_{gr} \quad \text{at} \quad y = 0; \tag{3a}$$

$$\lambda_{gr}\frac{\partial T_{pi}}{\partial x} = 0 \quad \text{at} \quad x = 0 \begin{cases} 0 \leqslant y \leqslant h_0 - R_0 \\ h_0 - R_0 \leqslant y < \infty \end{cases}; \tag{4a}$$

$$T_{pi} = 0 \quad \text{as} \quad x \to \infty,\ y \to \infty; \tag{5a}$$

$$T_{pi} = T_{gro} - T_{nat} \quad \text{for} \quad t \leqslant 0. \tag{6a}$$

The expression for $T_{nat} = T_{nat}(x, y, t)$ is given in [1, 4].

We solve Eq. (1a) by using a system of bipolar coordinates obtained by the conformal transformation [5]

$$x + iy = ci\,\mathrm{cth}[0{,}5(\alpha + i\beta)], \tag{8}$$

where

$$x = c\sin\beta/(\mathrm{ch}\,\alpha - \cos\beta);\ y = c\,\mathrm{sh}\,\alpha/(\mathrm{ch}\,\alpha - \cos\beta);\ c = \sqrt{h_0^2 - R_0^2}.$$

TABLE 1. Heat Flux (W/m^2) Calculated by Analytic Formulas and by Numerical Modeling on a Computer

t, h	0	20	40	60	80	100	120	140	160
Calculated by computer	438,6	167,8	135,4	120,7	111,8	105,7	101,1	97,6	94,9
By Eq. (12)	435.7	188,0	151,0	129,6	116,1	106,8	100,0	94,8	90,6
Relative error	0,7	8,5	11,4	7,3	3,8	1,1	1,1	2,9	4,5
By Eq. (16)	168,2	153,4	140,9	130,2	121,2	113,5	106,9	101,4	96,6
Relative error	61,7	8,6	4,1	7,9	8,4	7,4	5,7	3,8	1,9
By Eq. (18)	176,7	158,0	142,6	129,9	119,4	110,8	103,6	97,7	92,0
Relative error	57,1	5,9	5,3	7,6	6,8	4,8	2,4	0,1	2,1

Since the mapping (8) is conformal, the Laplacian operator in (1a) is invariant; i.e.,

$$\partial T_{pi}/\partial t = a_{gr}g^2(\partial^2 T_{pi}/\partial\alpha^2 + \partial^2 T_{pi}/\partial\beta^2). \quad (1b)$$

We rewrite the boundary conditions in the new coordinates in the form

$$\lambda_{gr} g\partial T_{pi}/\partial\alpha = -\alpha_1(T - T_{pi} - T_{nat}) - \lambda_{gr} g\partial T_{nat}/\partial\alpha \quad \text{for} \quad \alpha = \alpha_0; \quad (2b)$$

$$\lambda_{gr} g\partial T_{pi}/\partial\alpha = \alpha_{s0} T_{pi} \quad \text{for} \quad \alpha = 0; \quad (3b)$$

$$\lambda_{gr}\partial T_{pi}/\partial\beta = 0 \quad \text{for} \quad \beta = 0;\ \beta = \pi; \quad (4b)$$

where

$$\alpha_0 = \ln[h_0/R_0 + \sqrt{(h_0/R_0)^2 - 1}];\ g = (\operatorname{ch}\alpha - \cos\beta)c^{-1}.$$

Following [2] we linearize the variable coefficients in Eq. (1b) and boundary conditions (2b) and (3b) in order to obtain an expression suitable for practical calculations. It will be shown below that such a linearization gives satisfactory results in the calculation of heat losses.

After linearization Eqs. (1b)-(4b) take the form

$$\partial T_{pi}/\partial t = a^*_{gr}(\partial^2 T_{pi}/\partial\alpha^2 + \partial^2 T_{pi}/\partial\beta^2); \quad (1c)$$

$$\partial T_{pi}/\partial\alpha = \mathrm{Bi}_1(T - T_{pi} - T_{nat}) \quad \text{for} \quad \alpha = \alpha_0; \quad (2c)$$

$$\partial T_{pi}/\partial\alpha = \mathrm{Bi}_2 T_{pi} \quad \text{for} \quad \alpha = 0; \quad (3c)$$

$$\partial T_{pi}/\partial\beta = 0 \quad \text{for} \quad \beta = 0;\ \beta = \pi, \quad (4c)$$

where

$$\mathrm{Bi}_1 = \alpha_1 c/\lambda_{gr}\operatorname{ch}\alpha_0;\ \mathrm{Bi}_2 = \alpha_{s0}c/\lambda_{gr};\ a^*_{gr} = a_{gr}(1 + \operatorname{sh}2\alpha_0/4\alpha_0)c^{-2}.$$

The solution of problem (1c)-(4c), (6a) is found by the Koshlyakov—Grinberg method of finite integral transforms [6]. The kernel of the transformation with respect to the variable α for the Sturm—Liouville problem under consideration has the form

$$K_n(\alpha, \mu_n) = C_n^{-1}\left(\cos\mu_n\alpha + \frac{\mathrm{Bi}_2}{\mu_n}\sin\mu_n\alpha\right), \quad (9)$$

where the normalization factor C_n is

$$C_n = \frac{\alpha_0}{2}\left(1 + \frac{\mathrm{Bi}_2^2}{\mu_n^2}\right) + \frac{\sin 2\mu_n\alpha_0}{4\mu_n}\left(1 - \frac{\mathrm{Bi}_2^2}{\mu_n^2}\right) + \frac{\mathrm{Bi}_2\sin^2\mu_n\alpha_0}{\mu_n^2}. \quad (9a)$$

The eigenvalues of the problem μ_m are the positive roots of the characteristic equation

$$\operatorname{ctg}\mu_n\alpha_0 = (\mu_n^2 - \mathrm{Bi}_1\mathrm{Bi}_2)[\mu_n(\mathrm{Bi}_1 + \mathrm{Bi}_2)]^{-1}. \quad (9b)$$

Taking the integral transform in the interval $0 \leqslant \alpha \leqslant \alpha_0$ with the kernel (9), solving the representative equation, taking the inverse transform, and improving the convergence of the series by the Grinberg method [7], we obtain the solution of problem (1c), (2c)-(4c), (6a) in the form

$$T_{gr}(\alpha, \beta, t) = T_{gr} + \sum_{n=1}^{\infty}\{\tilde{T}_0\exp(-a^*_{gr}\mu_n^2 t) + a^*_{gr}A_n\int_0^t(T - T_{nat})\exp[-a^*_{gr}\mu_n^2(t-\tau)]\,d\tau - A_n(T - T_{nat})\mu_n^{-2}\}K_n(\alpha, \mu_n), \quad (10)$$

where the function $\tilde{T}_0 = \int_0^{\alpha_0} (T_{gr0} - T_{nat}) C_n K_n(\alpha, \mu_n) d\alpha$ is equal to zero (startup of pipeline after construction is completed or after a long shutdown) or $\tilde{T}_0 = Bi_1(T_0 - T_{nat0})(\mu_n^2 + Bi_2^2) \cdot \cos\mu_n\alpha_0 [\mu_n^2(\mu_n^2 - Bi_1Bi_2)]^{-1}$ (change of transfer conditions); T_{qu} is the temperature of the ground in a quasisteady thermal state, $T_{qu} = T_{nat} + Bi_1(1 + \alpha Bi_2)(T - T_{nat0})[Bi_1(1 + \alpha_0 Bi_2) + Bi_2]^{-1}$; $A_n = Bi_1(\mu_n^2 + Bi_2^2)(\mu_n^2 - Bi_1Bi_2)^{-1}\cos\mu_n\alpha_0$.

By using Eq. (10) the unsteady temperature distribution of the ground can be found for an arbitrary variation of the temperature of the medium being transferred $T = T(t)$ and the natural temperature distribution in the ground $T_{nat} = T_{nat}(x, t)$ for boundary conditions of the third kind at the pipe wall and the surface of the ground.

Using Fourier's law we determine the change in heat flux around the perimeter of the pipe from (10):

$$\bar{q} = \lambda_{gr} g \left.\frac{\partial T_{gr}}{\partial \alpha}\right|_{\alpha=\alpha_0} = \lambda_{gr} \frac{\operatorname{ch}\alpha_0 - \cos\beta}{c} \left\{ \delta c \frac{1 - \operatorname{ch}\alpha_0 \cos\beta}{(\operatorname{ch}\alpha_0 - \cos\beta)^2} + \right.$$

$$+ \frac{(T - T_{nat0}) Bi_1 Bi_2}{Bi_1(1 + \alpha_0 Bi_2) + Bi_2} + \sum_{n=1}^{\infty} \left[\tilde{T}_0 \exp(-a_{gr}^* \mu_n^2 t) + a_{gr}^* \int_0^t (T - \right.$$

$$\left. \left. - T_{nat0}) A_n \exp[-a_{gr}^* \mu_n^2 (t - \tau)] d\tau - (T - T_{nat0}) A_n \mu_n^{-2} \right] K_n'(\alpha_0, \mu_n) \right. \quad (11)$$

and its average over the perimeter

$$q = \frac{1}{2\pi R_0} \int_{-\pi}^{\pi} \frac{\partial T_{gr}}{\partial \alpha} d\beta = q_{qu} + \frac{\lambda_{gr}}{R_0} \sum_{n=1}^{\infty} [\tilde{T}_0 \exp(-a_{gr}^* \mu_n^2 t) +$$

$$+ a_{gr}^* \int_0^t (T - T_{nat0}) A_n \exp[-a_{gr}^* \mu_n^2 (t - \tau)] d\tau - (T - T_{nat0}) \mu_n^{-2}] K_n'(\alpha_0, \mu_n), \quad (12)$$

where δ is the gradient of the natural temperature distribution of the ground, and q_{qu} measures the heat losses of an oil line during quasisteady heat transfer:

$$q_{qu} = \lambda_{gr}(T - T_{nat0}) Bi_1 Bi_2 R_0^{-1} [Bi_1(1 + \alpha_0 Bi_2) + Bi_2]^{-1};$$

$$K_n'(\alpha_0, \mu_n) = (Bi_2^2 + 2Bi_1Bi_2 - \mu_n^2)(Bi_1 + Bi_2)^{-1} C_n^{-1}.$$

A solution more convenient for practical applications can be obtained by the Bubnov–Galerkin method [8]. We write the solution of problem (1b), (2b)-(4b), (6a) as the finite sum

$$T_{pi} = \psi(\alpha, \beta) + \sum_{n=1}^{N} C_n(t) \varphi_n(\alpha, \beta), \quad (13)$$

in which the functions $\psi(\alpha, \beta)$ and $\varphi_n(\alpha, \beta)$ are chosen so as to satisfy boundary conditions (2b)-(4b). Requiring that $R(\alpha, \beta, C_1, \ldots, C_n) = g^2(\alpha, \beta)(\partial^2 T_{pi}/\partial\alpha^2 + \partial^2 T_{pi}/\partial\beta^2) - a_{gr}^{-1}\partial T_{pi}/\partial t$ be orthogonal to all the functions $\{\varphi_n(\alpha, \beta)\}$ leads to the system of algebraic equations

$$\int_0^{\alpha_0} \int_{-\pi}^{\pi} R(\alpha, \beta, C_1, \ldots, C_n) \varphi_n(\alpha, \beta) d\beta d\alpha = 0 \quad (n = 1, 2, \ldots, N). \quad (14)$$

Limiting ourselves to the first term (N = 1) in (14) and determining $C_1(t)$, we obtain the solution of (1b), (2b)-(4b), (6a) in the form

$$T_{gr}(\alpha, \beta, t) = T_{qu} + B\alpha(\alpha_0 - \alpha)(\operatorname{ch}\alpha_0 + \cos\beta)(\operatorname{ch}\alpha_0 + \Phi)^{-1} -$$

$$- a_{gr}\gamma B\alpha(\alpha_0 - \alpha)(\operatorname{ch}\alpha_0 + \cos\beta)(\operatorname{ch}\alpha_0 + \Phi)^{-1} \int_0^t \theta(\tau) \exp[-a_{rp}\gamma(t - \tau)] d\tau, \quad (15)$$

where

$$B = 2{,}5 Bi_1(2 + \alpha_0 Bi_2)\alpha_0^{-2}[Bi_1(1 + \alpha_0 Bi_2) + Bi_2]^{-1};$$

$$\gamma = 10\alpha_0^{-2}c^{-2} + \Phi c^{-2}(\operatorname{ch}\alpha_0 + \Phi)^{-1}; \ \Phi = (5\alpha_0 - 3\operatorname{sh}2\alpha_0) [2\operatorname{ch}\alpha_0(4\alpha_0 + \operatorname{sh}2\alpha_0) - 8\operatorname{sh}\alpha_0];$$

$\theta = T - T_0$ for a change in the transfer conditions, and $\theta = T - T_{nat0}$ for starting up the pipeline. Taking account of (15) the expression for heat losses takes the form

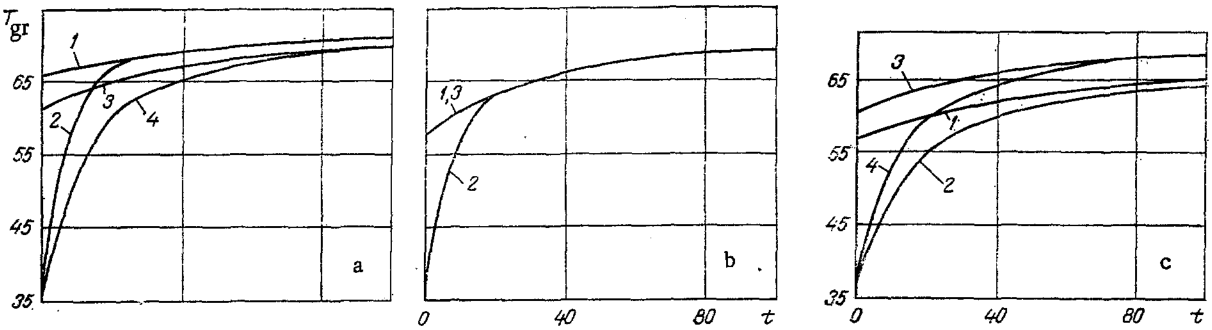

Fig. 1. Change in temperature of the ground at points with coordinates $\alpha = 0.45\pi$, $\beta = \pi$ (a), $\alpha = 0.45\pi$, $\beta = 0.5\pi$ (b), and $\alpha = 0.45\pi$, $\beta = 0$ (c): 1) by Eq. (15); 2) numerical solution; 3) by Eq. (17); 4) by Eq. (10).

$$q = q_{qu} + \frac{\lambda_{gr} B\theta\alpha_0 \operatorname{ch}\alpha_0}{R_0(\operatorname{ch}\alpha_0 + \Phi)} - a_{gr}\gamma \frac{\lambda_{gr} B\alpha_0 \operatorname{ch}\alpha_0}{R_0(\operatorname{ch}\alpha_0 + \Phi)} \int_0^t \theta \exp[-a_{gr}\gamma(t-\tau)]\, d\tau. \tag{16}$$

The solution of the linearized equations (1c), (2c)-(4c), (6a) is found in a similar way by the Bubnov—Galerkin method:

$$T_{gr}(\alpha, \beta, t) = T_{qu} + B\theta\alpha(\alpha_0 - \alpha) - 10a_{gr}^* B\alpha(\alpha_0 - \alpha) \int_0^t \theta \exp[-10a_{gr}^*(t-\tau)]\, d\tau, \tag{17}$$

$$q = q_{qu} + \theta B\alpha_0 R_0^{-1} - 10a_{gr}^* B\alpha_0 R_0^{-1} \int_0^t \theta \exp[-10a_{gr}^*(t-\tau)]\, d\tau. \tag{18}$$

In the limiting case as $t \to \infty$ the solutions (10)-(12), (15)-(18) correspond to steady heat exchange of an underground pipeline with the surrounding medium [9].

4. To estimate the accuracy of the solutions obtained above, calculations made with Eqs. (10), (12), (15)-(18) were compared with the results of a numerical integration of Eqs. (1)-(6) for a stepwise change $\Delta T = 40°C$ of the temperature of the medium being transferred, using the values $R_0 = 0.7$ m, $h_0 = 1.7$ m, $\lambda_{gr} = 1$ W/m·deg, $\alpha_1 = 500$ W/m²·deg, $\alpha_{S_0} = 10$ W/m²·deg, and $T_{nat_0} = 9.3°C$.

The temperatures calculated at various points in the ground around the pipeline are shown in Fig. 1. Analysis shows that there is better agreement between results calculated by the analytic formulas with those from numerical modeling on a computer at points on the level of the pipeline axis ($\beta = 0.5\pi$). At points above and below the pipe the differences between the values calculated by computer and by Eqs. (10) and (17) are considerably greater, reaching 6°C. The temperatures of the ground calculated by Eqs. (10) and (17) are higher than the computer values for points under the pipe ($\beta = \pi$) and lower at points above the pipe ($\beta = 0$), with these differences having nearly the same absolute value. This is related to the nature of the linearization of the coefficient $g(\alpha, \beta)$ in (1b). The approximate solution (15) of nonlinearized equation (1b) gives better agreement with the computer calculation than (17) does. The error in (15) and (17) at zero time results from retaining only the first term of the sum in (13).

The calculated heat losses of an underground pipeline q are shown in Table 1. The differences between values of q calculated by Eq. (13) and by computer are less than 10%. The error in Eqs. (16) and (18) is close to 10% for times $t \geqslant 20$ h. The total time of the study of the unsteady process was more than a month. For small values of the time ($t < 20$ h) the errors of (16) and (18) reach 60%. Therefore, for such times the heat losses must be determined by Eq. (13); for large times the heat losses can be determined by (18), since it gives the same accuracy and is more convenient than (13) and (16).

NOTATION

T_{gr}, temperature of ground; T_{nat}, temperature of ground in natural state; T_{gr_0}, initial distribution of ground temperature; T_{nat_0}, ground temperature at level of pipeline axis in natural thermal state; T, temperature of medium being transferred; T_0, temperature of medium being transferred at zero time; T_{ae}, equivalent air temperature; a_{gr}, λ_{gr}, thermal diffusivity and thermal conductivity of ground; α_1, coefficient of heat transfer from medium being transferred to pipe wall; α_{s0}, generalized coefficient of heat transfer from surface to atmosphere; R_0, pipe radius; h_0, depth of pipe axis below ground; t, time; x, y, Cartesian coordinates; α, β, bipolar coordinates.

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METHOD OF COMPUTING THE HEAT INFLUXES OVER ELECTRICAL CABLES IN CRYOGENIC SYSTEMS

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A method is proposed for computing the heat influx over electrical cables taking account of the dependence of the thermal conductivity of copper on the temperature and radiative heat exchange of the cable and the surrounding surfaces.

One of the fundamental requirements imposed on cryogenic systems is a minimum of external heat influx.

At the same time, a unit of temperature sensors is usually required for checking out and controlling the operation of the system elements. The heat influx over the cables can exert a considerable influence on both the system characteristics and on the readings of the sensors themselves because of the high thermal conductivity of copper, especially at temperatures below 30°K.

Meanwhile, it is usually customary to consider the heat influx over the conductors comprising the cable as though over rods heat-insulated from the side surface, with a constant magnitude of the thermal conductivity, which results in a multiple reduction of the true heat influx, as experiments have shown. As a rule, the low-temperature elements of cryogenic systems are in a vacuum; hence, heat flux by radiation from the surrounding walls proceeds to the side surface of the cable. If Joule heating is neglected because of the smallness of the measuring current through the sensor, the heat-transfer differential equation for the cable can be written in the form

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